

ANOVA F test analysis of variance

1 Calculate mean of each population :-

$$\left. \begin{aligned} m_1 &= \frac{1}{3} (1+2+5) = 2.67 \\ m_2 &= \frac{1}{3} (2+4+2) = 2.67 \\ m_3 &= \frac{1}{3} (2+3+4) = 3 \end{aligned} \right\} \text{equal by chance}$$

data sets

D ₁	D ₂	D ₃
1	2	2
2	4	3
5	2	4

K → number of populations
K = 3

calculate grand mean m_0 :-

$$m_0 = \frac{1}{3} (2.67 + 2.67 + 3) = 2.78$$

2 calculate sum of Squares (SS) :-

$$\begin{aligned} SS_{\text{within}} &= \sum_i (x_{1i} - m_1)^2 + \sum_i (x_{2i} - m_2)^2 + \sum_i (x_{3i} - m_3)^2 \\ &= [(1 - 2.67)^2 + (2 - 2.67)^2 + (5 - 2.67)^2] + [(2 - 2.67)^2 + (4 - 2.67)^2 + (2 - 2.67)^2] \\ &\quad + [(2 - 3)^2 + (3 - 3)^2 + (4 - 3)^2] = 13.34 \end{aligned}$$

$$SS_{\text{Total}} = \sum (x - m_0)^2$$

$$\begin{aligned} &= (1 - 2.78)^2 + (2 - 2.78)^2 + (5 - 2.78)^2 + (2 - 2.78)^2 + (4 - 2.78)^2 + \\ &\quad (2 - 2.78)^2 + (2 - 2.78)^2 + (3 - 2.78)^2 + (4 - 2.78)^2 = 13.6 \end{aligned}$$

$$SS_{\text{between}} = SS_{\text{Total}} - SS_{\text{within}} = 13.6 - 13.34 = 0.23$$

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N : total number of sample = $3+3+3=9$, $K=3$

$$S_w^2 = \frac{SS_{within}}{N-K} = \frac{13.34}{9-3} = 2.22$$

$$S_B^2 = \frac{SS_{Between}}{K-1} = \frac{0.23}{3-1} = 0.115$$

$$F = \frac{S_B^2}{S_w^2} = 0.05$$

if $F > 1$, then difference between hypothesis is big
then reject Null hypothesis

if $F < 1$, then difference is small and not reject Null hypothesis.

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